CHAPTER 4: ESTIMATING METHODS

(Updated: December 2015)

The NIPA measures are built up from a wide range of source data using a variety of estimating methods. Each NIPA component is derived using a specific methodology—that is, source data and estimating methods—that progresses from the advance quarterly estimate through the comprehensive NIPA revision.

The methodologies used to prepare the various NIPA estimates are periodically changed in order to incorporate improvements in the source data or in the estimating methods.[1]

- Over time, source data may emerge or disappear, so new source data must be identified and evaluated, and estimating methods must be adapted accordingly.
- Advances in statistical techniques or in other aspects of estimation must be evaluated for adoption into the methodology.

[1] Substantive changes to NIPA methodologies are documented in BEA's monthly *Survey of Current Business*.

- As the U.S. economy evolves, the methodologies must be updated to ensure that the estimates continue to provide a reliable and relevant picture of transactions and transactors in the economy.

The examples provided in this chapter are simplified in order to illustrate the basic estimating concepts and calculations. In practice, the procedures used for deriving the NIPAs involve complex statistical techniques that are designed to ensure consistency across the entire time series for a given estimate and between interrelated estimates.

Current-Dollar Estimates

For most NIPA components, the current-dollar, or nominal, estimates are derived from source data that are "value data," which reflect the product of quantity and price. For the estimates that are not derived from value data, separate quantity data and price data must be combined. For example, an estimate of expenditures on new autos may be calculated as the number of autos sold times expenditure per auto (at transaction prices—that is, the average list price with options adjusted for transportation charges, sales taxes, dealer discounts, and rebates). An estimate of wages may be calculated as employment times average hourly earnings times average hours worked, and an estimate of interest received may be calculated as the stock of interest-bearing assets times an effective interest rate. (The NIPA current-dollar estimates are expressed at annual rates; see the appendix to this chapter.)

Adjustments to the source data

BEA makes three general types of adjustments to the source data that are incorporated into the NIPA estimates. The first consists of adjustments that are needed so that the data conform to appropriate NIPA concepts and definitions. For example, Internal Revenue Service data from corporate tax returns include estimates of depreciation, but these estimates are based on historical-cost valuation and on tax service lives. BEA must adjust these estimates to the NIPA definition of depreciation—consumption of fixed capital—which is based on current-cost valuation and economic service lives.

The second type of adjustment involves filling gaps in coverage. For example, one of the primary sources for the quarterly estimates of the change in private inventories component of GDP is the Census Bureau's monthly survey of wholesale trade. However, this source does not cover inventories of nonmerchant wholesalers (wholesalers that do not take title to the goods they sell). Thus, the survey data must be augmented by separate BEA estimates for the change in the inventories of these wholesalers.

The third type of adjustment involves time of recording and valuation. For example, in the NIPAs (as in BEA's international transactions accounts), imported goods are valued at "foreign port value"—that is, the value at the point of exportation to the

United States. The source data on imports of goods from Canada, which the Census Bureau receives in a bilateral data exchange with Canada, are often valued at the point of manufacture; thus, BEA must adjust these data to foreign port value by adding the cost of transporting these goods within Canada from the point of manufacture to the point of export to the United States.

In addition, source data must occasionally be adjusted to account for special circumstances that affect the accuracy of the data. For example, the monthly current employment statistics are collected in the middle of the month, which is assumed to represent conditions during the entire month. Thus, these source data may need to be adjusted if a significant event, such as a blizzard that blankets much of the eastern United States, occurs during that period.

Seasonal adjustment

Quarterly and monthly NIPA estimates are seasonally adjusted at the detailed-series level when the series demonstrate statistically significant seasonal patterns. For most of the series that are seasonally adjusted by the source agency, BEA adopts the corresponding seasonal adjustment factors. Seasonal adjustment removes from the time series the average effect of variations that normally occur at about the same time and in about the same magnitude each year—for example, the effect of weather or holidays. After seasonal adjustment, trends, business cycles, and other movements in the time series stand out more clearly.

Many of the data used by BEA to estimate GDP are seasonally adjusted by the source data agencies, and BEA seasonally adjusts some of those that are not; other source data cannot be seasonally adjusted with conventional methods, such as the X-12 ARIMA process, until the time series is sufficiently long (that is, at least 5 years) to adequately capture seasonal trends.[2]

In general, the seasonal adjustment techniques used by BEA and its source data agencies successfully remove seasonal patterns from the estimates.[3] However, for a variety of reasons, including differences between monthly and quarterly seasonal patterns and the aggregation of data from different sources, residual seasonality may remain. BEA and its source data agencies regularly review and update seasonal adjustment procedures to adjust any source data not previously adjusted and to address residual

[2] X-12 ARIMA is a software program developed by the Census Bureau to identify and remove seasonal effects from a time series; for more information, see the Census Bureau website at www.census.gov. In cases where the series lacks a sufficient time span to derive seasonal factors, BEA often uses smoothing techniques such as moving averages to reduce seasonality in the data.

[3] This is evident in the strong seasonality apparent in the non-seasonally adjusted estimates that were produced by BEA until they were discontinued in 2008.

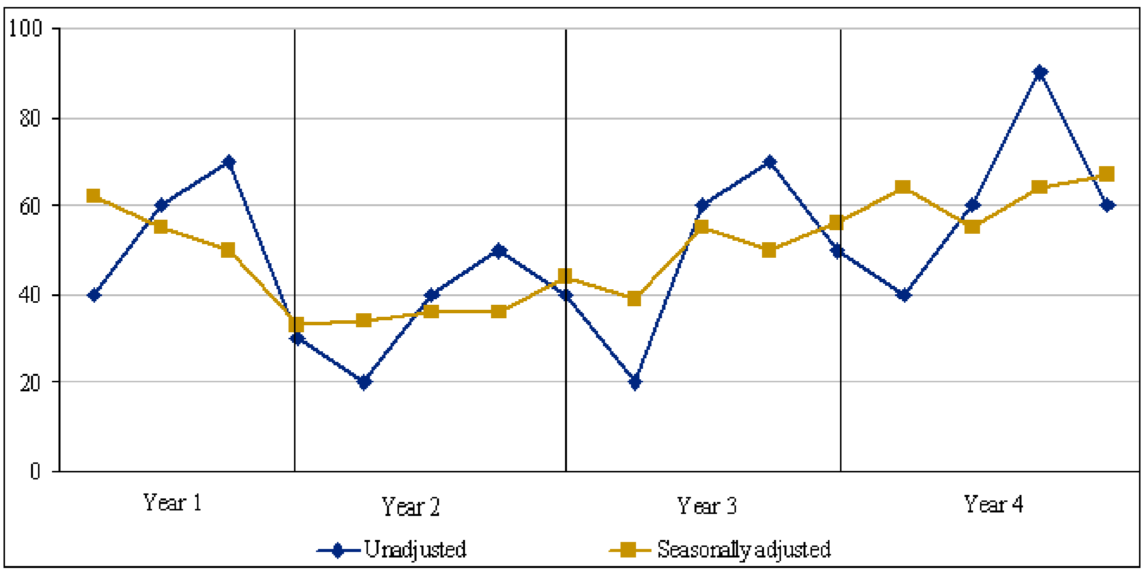
seasonality emerging over time, and BEA is engaging in research to identify and address additional sources of residual seasonality.[4]

Table 4.1 and chart 4.1 illustrate the effects of seasonally adjusting a series that has a significant seasonal pattern. The unadjusted series shows a pattern of consistent strength in the second and third quarters and corresponding weakness in the first and fourth quarters. The series is adjusted by calculating seasonal adjustment factors and dividing them into the unadjusted values for the appropriate quarter. As necessary, further adjustments are then made to ensure that the seasonally adjusted quarterly values sum to the annual total for that year.

Table 4.1—Simplified Example of Seasonal Adjustment

	Unadjusted					Seasonally adjusted			
Quarter	I	II	III	IV	Total	I	II	III	IV
Year									
1	40	60	70	30	200	62	55	50	33
2	20	40	50	40	150	34	36	36	44
3	20	60	70	50	200	39	55	50	56
4	40	60	90	60	250	64	55	64	67

Chart 4.1—Illustration of Seasonal Adjustment



[4] For more information on residual seasonality and BEA's associated research agenda, see Brent R. Moulton, "Residual Seasonality in GDP and GDI: Findings and Next Steps," *Survey of Current Business* 96 (July 2016): 1-6 and see Stephanie H. McCulla and Shelly Smith, "The 2016 Annual Update of the National Income and Product Accounts," *Survey of Current Business* 96 (August 2016): 1-31.

Two seasonal adjustment strategies are commonly used: Regular seasonal adjustments use seasonal factors that are based on the factors for prior years, and concurrent seasonal adjustments are redone each period (quarter or month) using all the estimates up to and including the current period to calculate the seasonal factor. Because seasonal patterns may change over time, complex statistical techniques have been developed to seasonally adjust time series data. The most widely used method is the Census Bureau's X-12 ARIMA program, which uses a statistical analysis to calculate how the seasonal pattern of a time series has changed recently and how it might be expected to change further over the coming year.

Moving average

A moving average is a calculation that is used to smooth a data series that is characterized by volatile short-term fluctuations. As a result, trend and cyclical movements in the smoothed series will be more apparent, and the series can be better used as an indicator for interpolation and extrapolation (see below).

Table 4.2 illustrates the smoothing effects of a three-quarter moving average on a volatile series. The simple moving average is calculated by summing the value in a given quarter and in the preceding two quarters and dividing by 3 (in year 1:III, $(90.0 + 120.0 + 100.0)/3 = 103.3$). A weighted moving average is calculated by assigning greater weight to the time periods that are deemed more relevant. In this example, the weighted moving average is calculated by weighting the current quarter at 50 percent, and the two preceding quarters at 25 percent each (in year 1:III, ($(90.0 \times 0.50) + (120.0 \times 0.25) + (100.0 \times 0.25) = 100.0$).

Table 4.2—Example of Moving-Average Calculation

Time period	Original series	Simple moving average	Weighted moving average
Year 1:I	100.0	…..	…..
Year 1:II	120.0	…..	…..
Year 1:III	90.0	103.3	100.0
Year 1:IV	150.0	120.0	127.5
Year 2:I	170.0	136.7	145.0
Year 2:II	100.0	140.0	130.0
Year 2:III	150.0	140.0	142.5
Year 2:IV	120.0	123.3	122.5

Alternatively, a "centered" three-quarter moving average could be calculated, in which the quarterly value is the average of the value in the current quarter and the values in the preceding quarter and in the following quarter. This would have the effect of shifting the moving-average series back one quarter (in the example, the value of the centered moving average would be 103.3 in year 1:II, and so forth through 123.3 in year 2:III).

Best level and best change

Source data are incorporated into the NIPA estimates on either a "best-level" or a "best-change" basis. Best level provides the most accurate value for an economic statistic at a specified point in time using the best available source data. For example, in a comprehensive update of the NIPAs, data from the quinquennial economic census are incorporated into the estimates on a best-level basis.

However, it is not practical to revise the entire NIPA time series every time new or revised source data become available. Thus, these data are often initially introduced into the estimates on a best-change basis. Best change provides the most accurate measure of the period-to-period movement in an economic statistic using the best available source data. In an annual update of the NIPAs, data from the annual surveys of manufacturing and trade are generally incorporated into the estimates on a best-change basis. In the current quarterly estimates, most of the components are estimated on a best-change basis from the annual levels established at the most recent annual update.

In table 4.3, the original series of source data (column 1) has been revised as shown by the best-level series (column 3). In the example, the level of the series has been revised up in all years, perhaps reflecting a change in definition, and the percent changes in the series have been revised to incorporate new statistical information. In an annual NIPA revision, the revised levels of the source data cannot be fully incorporated, because annual updates only cover the 3 most recent years.[5] As can be seen in this example, incorporating the revised best-level series only for years 2–4 would result in a discontinuity between the unrevised estimate for year 1 (100.0) and the revised estimate for year 2 (115.0) (a 15.0-percent increase rather than the 10.6-percent increase indicated by the source data). To avoid this problem, the revised source data are instead incorporated on a best-change basis—that is, a new best-change series is created by beginning with the value in the unrevised year 1 (100.0) and applying the percent changes in the best-level series (column 4). As a result, the level of the new series (column 5) is kept consistent with the level of the earlier nonrevised year, while the percent changes in the new series (column 6) fully reflect the new statistical information that was incorporated into the source data. In the next comprehensive update, the revised best-levels would be incorporated into the NIPA estimates.

Table 4.3—Simplified Example of "Best Level" and "Best Change"

Year	Original series [billions of dollars] (1)	Percent change in original series (2)	Revised ("best-level") series [billions of dollars] (3)	Percent change in best-level series (4)	Revised ("best-change") series [billions of dollars] (5)	Percent change in best-change series (6)
1	100.0	…..	104.0	…..	100.0	…..
2	110.0	10.0	115.0	10.6	110.6	10.6
3	120.0	9.1	124.0	7.8	119.2	7.8
4	130.0	8.3	136.0	9.7	130.8	9.7

Interpolation and extrapolation using an indicator series

Generally, monthly or quarterly source data are not as comprehensive or as reliable as annual source data (and, similarly, annual source data are not as comprehensive or as reliable as quinquennial source data). Thus, for some estimates, the more frequent but less comprehensive source data may be used as an indicator of the movements of the component series rather than as a measure of the absolute levels of the series. Specifically, for the periods for which annual estimates are available and the quarterly estimates must be forced to average to these annual totals, the quarterly pattern is estimated by *interpolation*. For the periods not yet covered by annual estimates (such as the current quarter), the quarterly estimates are made by *extrapolation*.

[5] Starting in 2010, BEA instituted a "flexible" approach to annual updates that allows for the incorporation of improvements in methodology to be introduced and for the extension of the 3-year revision period to earlier periods; see "BEA Briefing: Improving BEA's Accounts Through Flexible Annual Revisions," *Survey* 88 (June 2008): 29–32.

The use of an indicator series to estimate a component is illustrated in table 4.4. We begin with a value of \$200 (annual rate) for the fourth quarter of year 1 (this value was determined by the preceding year's calculation) and a value of \$220 for the year 2 (this value was determined from an annual data source). Because the detailed source data are not available on a quarterly basis, the estimates for the quarters of year 2 are interpolated using an indicator series whose movements are deemed to approximate those of the component series. In this simplified example, the interpolation of the quarterly values is accomplished by calculating a time series that begins with the established value (\$200) for the fourth quarter of year 1 and progresses through the four quarters of year 2 at the same rate of change as the indicator series: for year 2:I, \$200 + (\$200 × 0.20) = \$240; for year 2:II, \$240 + (\$240 × –0.167) = \$200; and so forth. As necessary, the calculated series is then adjusted to ensure that the average for the four quarters of year 2 is equal to the established annual value for year 2: for year 2:I, \$240 × (\$220 ⁄ \$240) = \$220; for year 2:II, \$200 × (220 ⁄ \$240) = \$183.3; and so forth.

Similarly, the estimates for the quarters of the current year, year 3, can be calculated by extrapolating the value for the fourth quarter of year 2 using the percent change in the values for the indicator series as they become available: for year 3:I, \$256.7 + (\$256.7 × 0.20) = \$308.0.

Table 4.4—Simplified Example of Estimation Using an Indicator Series

Time period	Established value	Indicator series	Percent change in indicator series	Calculated value	Adjusted series
Year 1:IV	200	25	…..	…..	
Year 2:I		30	20.0	240	220.0
Year 2:II		25	–16.7	200	183.3
Year 2:III		30	20.0	240	220.0
Year 2:IV		35	16.7	280	256.7
Year 2: Total	220			240	220.0
Year 3:I		42	20.0		308.0

Over time, BEA has used a number of different statistical techniques for interpolation of NIPA time series. Currently, BEA is using a procedure known as the "proportional Denton method" or "quadratic minimization." In its most common application, this approach interpolates series by minimizing the sum of the squared differences of the ratios of the interpolated series and the indicator series. Formally, the interpolation is estimated by the following optimization problem:

$$\min_{x_t} \sum_{t=2}^{4N} \left(\frac{x_t}{z_t} - \frac{x_{t-1}}{z_{t-1}} \right)^2, \qquad s.t. \sum_{t=1}^{4} x_t = A_1, \ldots \sum_{t=4N-3}^{N} x_t = A_N$$

,

where z is the indicator series, x is the interpolated series, A are the annual controls that the interpolated series must sum to, and N is the number of years for the interpolation. This example shows an annual-to-quarterly interpolation. The same method can also be used for annual-to-monthly and quarterly-to-monthly interpolation.[6]

Three special estimation methods

In certain cases where primary source data are not available, one or more of the following special methods—commodity flow, retail control, or perpetual inventory—may be used to estimate values.

Commodity-flow method

The commodity-flow method is generally used to derive estimates in economic census years for various components of consumer spending, equipment and software, and the commodity detail for state and local government consumption expenditures and gross investment. An abbreviated form of this method is used to prepare estimates of investment in equipment in nonbenchmark years, and an even more abbreviated form is used to prepare the current quarterly estimates of investment in equipment.[7]

The commodity-flow method begins with estimates of the domestic output or domestic sales of a commodity valued in producers' prices.[8] Then, estimates of the domestic supply of that commodity—the amount that is available for domestic consumption—are prepared by adding imports and by subtracting exports and inventory change. Next, the domestic supply of the commodity is allocated among domestic purchasers—that is, persons, business, and government. Finally, the estimates are converted to purchasers' prices.[9]

The commodity-flow method is illustrated in table 4.5. First, domestic shipments—the value of shipments of the commodity produced by domestic firms at producers' prices—are converted to net supply, by adding imports and subtracting exports, government purchases, and change in inventories (a positive change in inventories reduces net supply and a negative change in inventories raises net supply) (in

[6] See Baoline Chen and Stephen H. Andrews, "An Empirical Review of Methods for Temporal Distribution and Interpolation in the National Accounts," *Survey* 88 (May 2008): 31–37.
[7] For more information on using the commodity-flow method to prepare the estimates of investment in equipment, see "Chapter 6: Private Fixed Investment," pages 10–11.
[8] Producers' prices are the prices received by producers for the goods and services they sell. These prices include sales and excise taxes but exclude domestic transportation costs and trade margins. Trade margins, or markups, reflect the value added by wholesalers and retailers in the distribution of a commodity from producers to final purchasers.
[9] Purchasers' prices are the prices paid by intermediate and final purchasers for the goods and services they buy. These prices are equal to producers' prices plus domestic transportation costs and trade margins.

the example, $100 + $40 – $10 – $5 – $5 = $120). Portions of the net supply are then allocated among business intermediate purchases and consumer spending. This allocation may be based on relationships from the most recent economic census or on information from other sources (such as spending by consumers as determined by the retail control method). In this example, it is assumed that one-fourth of net supply is allocated to business intermediate purchases and one-sixth to personal consumption expenditures. Investment in equipment (prior to adjustments for transportation costs and wholesale and retail trade margins) is then computed as net supply less business intermediate purchases and consumer spending (in the example, $120 – $30 – $20 = $70). This estimate is then converted to purchasers' prices by adding domestic transportation costs and trade margins ($70 + $5 + $10 = $85).

Table 4.5—Simplified Example of Commodity-Flow Calculation

Factors for commodity flow	Values
Output (shipments)	100
Plus: Imports	40
Less: Exports	10
Government purchases	5
Inventory change	5
Equals: Net supply	120
Less: Business intermediate purchases	30
Personal consumption expenditures	20
Equals: Private fixed investment (producers' prices)	70
Plus: Domestic transportation costs	5
Trade margins	10
Equals: Private fixed investment (purchasers' prices)	85

Retail control method

The retail control method uses retail and food services sales data, compiled by the Census Bureau, to estimate annual, quarterly, and monthly consumer spending on most consumer goods in nonbenchmark years. In these years, the estimate of total personal consumption expenditures (PCE) on most goods is derived by extrapolation from the benchmark year using a retail control total of sales by most kinds of business from the Census Bureau's monthly and annual surveys.

In general, product-based data on consumer spending are not available in nonbenchmark years, so the estimates for the detailed PCE categories are prepared by extrapolation using estimates of retail sales by corresponding product lines that, in turn, are based on retail sales by kind of business and on commodity sales data from the most recent quinquennial economic census.[10] Then, the extrapolated estimates are adjusted proportionately so that their sum is equal to that for total PCE.[11]

The retail control method is illustrated in table 4.6. First, the PCE control total for year 2 is derived by extrapolation, using the change in the retail control total from year 1 to year 2 (89 × (120 ⁄ 100) = 106.8).

In year 1, a benchmark year, information from the economic census is available to break sales down into product lines (and to corresponding PCE categories) for each kind of business (such as “grocery stores”). In year 2, the annual survey of retail sales provides data on sales by kind of business but not on sales by individual product lines. In order to estimate sales by product line for year 2, the product-line distribution of sales from year 1 is applied to the sales by kind of business for year 2 (for kind of business A, 0.2 × 60 = 12 for product line 1, and 0.8 × 60 = 48 for product line 2). Total sales for each product line are then computed by summing across all kinds of business (for product line 1, 12 + 36 = 48; and for product line 2, 48 + 24 = 72).

The retail sales product lines in the Census Bureau’s data and the PCE categories in the NIPAs do not always match (in the example, product line 1 at 44 is larger than PCE category 1 at 33). Thus, the retail sales data are used to extrapolate the PCE estimates for year 2 (for product line 1, 33 × (48 ⁄ 44) = 36). Finally, the PCE category estimates must be adjusted so they sum to the PCE control total for year 2 (for product line 1, the adjusted estimate for year 2 is 36 × (106.8 ⁄ 108) = 35.6).

[10] The estimates for some PCE categories, such as consumer purchases of new and used motor vehicles and of motor vehicle fuels, are prepared independently.
[11] For more information on using the retail control method to prepare the PCE estimates, see “Chapter 5: Personal Consumption Expenditures,” page 9.

Table 4.6—Simplified Example of Retail Control Calculation

	Year 1 (economic census)	Product ratios in year 1	Year 2 (annual survey)	Year 2 (calculated values)
Retail control total	100		120	
PCE control total	89			106.8
Retail sale data:				
Kind of business A	40		60	
Product line-1	8	0.2		12
Product line-2	32	0.8		48
Kind of business B	60		60	
Product line 1	36	0.6		36
Product line 2	24	0.4		24
Product-line sales:				
Line 1	44			48
Line 2	56			72
PCE sales data:				
Category 1	33			
Category 2	56			
PCE (summed by category)				108
Category 1				36
Category 2				72
PCE adjusted				106.8
Category 1				35.6
Category 2				71.2

Perpetual inventory method

The perpetual inventory method is used to indirectly derive historical-cost and constant-dollar estimates of net stocks of fixed assets, which, in turn, are used in deriving the NIPA estimates of consumption of fixed capital.[12] For each type of good, the perpetual inventory method calculates the net stock in each year as the cumulative value of gross investment through that year—including both new investment and net purchases of used assets (in order to capture shifts in ownership across NIPA sectors)—less the cumulative value of depreciation through that year. A variation of this method that omits depreciation is used to calculate the stocks of private inventories.

[12] Current-cost net stocks and current-cost depreciation (consumption of fixed capital) are derived by converting the corresponding constant-dollar estimates to the prices of the current period.

The perpetual inventory method is illustrated in table 4.7 (in this example, it is assumed that asset prices do not change over the course of the year). In year 1, the estimates of the beginning-of-year stocks for two types of assets, A and B, are equal to the end-of-year stocks for the preceding year.[13] For asset A, the end-of-year stock in year 2 is equal to the beginning-of-year stock in year 2 plus the value of investment in asset A during the year minus the value of depreciation during that year ($110 + $20 – $11 = $119).

Table 4.7—Simplified Example of Perpetual Inventory Calculation

	Asset A	Asset B	Total capital stock
Year 1:			
Beginning-of-year stock	100	50	150
Plus: Investment	20	10	
Minus: Depreciation	10	10	
Equals: End-of-year stock	110	50	160
Year 2:			
End-of-preceding-year stock	110	50	160
Plus: Investment	20	5	
Minus: Depreciation	11	10	
Equals: End-of-year stock	119	45	164

[13] The estimates of capital stock are very long time series, so virtually all assets currently in existence have been valued since they were produced.

Quantity and Price Estimates

Estimates for all of the NIPA aggregates and components are presented in current dollars. Changes in current-dollar estimates measure the changes in the market values of goods or services that are produced or sold in the economy. For many purposes, it is necessary to decompose these changes into price and quantity components. In the NIPAs, prices and quantities are expressed as index numbers with the reference year—at present, the year 2009—equal to 100. For selected series, quantities—or "real" (inflation adjusted) measures—are also expressed in chained (2009) dollars. (Period-to-period changes in quantities and prices are expressed as percent changes at annual rates; see "Statistical Tools and Conventions" in the appendix to this chapter.)

BEA prepares quantity estimates for GDP and its product-side components and for a few other aggregates and components. (For an illustration of the calculation of these estimates from a set of quantity and price information, see "Calculation of Output and Price Indexes" in the appendix to this chapter.)

Estimates for detailed components

For the detailed NIPA components, the quantity estimates are prepared using one of three methods—deflation, quantity extrapolation, or direct valuation—depending on the availability of source data. The quantity estimates are expressed as real values with 2009 (at present) as the reference year.

Deflation. Because the source data available for most components of GDP are measured in dollars rather than in units, the quantities of most of the detailed components are obtained by deflation. For deflation, quantities are calculated by dividing the current-dollar value of the component by an "appropriate" price index (with the reference-year value set to 100).[14]

$Q_t = (p_t q_t)/(p_t/p_o)$, where p_t and q_t are observed prices and quantities in the current year and p_o is the observed price in the reference year.

Thus, for example, if the current-dollar value for the component series is $14 in 2010 and the appropriate price index is 112 in 2010, then the quantity estimate for the component series in 2010 is ($14 ⁄ (112 ⁄ 100)), or $12.50.

The price indexes used for deflation are generally adjusted for changes in characteristics or quality as described in the appendix to this chapter.

Quantity extrapolation. The other two methods are similar in that they both are derived using quantity data. Quantity extrapolation is used when a quantity indicator series is available that approximates the movements of the component series. In this

[14] A price index is appropriate if its definition and coverage closely match those of the series being deflated.

method, the quantity estimate is obtained by using the indicator series to extrapolate from the reference-year value.

$Q_t = p_o q_o + ((p_o q_o) \times ((q_t' - q_o')/q_o'))$, where q' represents the quantity indicator series.

For example, if the dollar value of the component series is \$10 in 2009 and the quantity indicator series shows an increase of 25 percent in 2010, then the quantity estimate for the component series in 2010 is (\$10 + (\$10 × 25/100)), or \$12.50.

Direct valuation. Direct valuation is used when physical quantity data and price data are available. In this method, the quantity estimate is obtained by multiplying the reference-year price by the actual quantity data for the current year.

$Q_t = p_o q_t$.

For example, if the price of the detailed component is \$.50 per unit in 2009 and the quantity measure is 20 units in 2009 and 25 units in 2010, then the quantity estimate for the component series in 2010 is (\$.50 × 25), or \$12.50.

Estimates for NIPA aggregates

The fundamental problem confronting the efforts to adjust GDP and other aggregates for inflation is that there is not a single inflation number but rather a wide spectrum of goods and services with prices that are changing relative to one another over time. The index numbers for the individual components can be combined statistically to form an aggregate index, but the method of aggregation that is used affects the movements of the resulting index.

In the NIPAs, the annual changes in quantities and prices are calculated using a Fisher formula that incorporates weights from 2 adjacent years.[15] For example, the 2011–2012 change in real GDP uses prices for 2011 and 2012 as weights, and the 2011–2012 change in prices uses quantities for 2011 and 2012 as weights. These annual changes are "chained" (multiplied) together to form time series of quantity and price indexes. Quarterly changes in quantities and prices are calculated using a Fisher formula that incorporates weights from two adjacent quarters; quarterly indexes are adjusted for consistency to the annual indexes before percent changes are calculated.

The Fisher index (Q_t^F) for calculating real GDP (and other aggregate measures of output and expenditures) in year *t* relative to its value in the previous year *t-1* is

[15] This formula is named after Irving Fisher, who originally developed this index to more accurately measure quantity and price changes over time.

$$Q_t^F = \sqrt{\frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_t q_{t-1}}},$$

where the *p*'s and *q*'s represent prices and quantities of detailed components in the 2 years.

Because the first term in the Fisher formula is a Laspeyres quantity index (Q_t^L), or

$$Q_t^L = \frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}},$$

and the second term is a Paasche quantity index (Q_t^P), or

$$Q_t^P = \frac{\sum p_t q_t}{\sum p_t q_{t-1}},$$

the Fisher formula can also be expressed for year *t* as the geometric mean of these indexes as follows:

$$Q_t^F = \sqrt{Q_t^L \times Q_t^P}.$$

The percent change in real GDP (and in other measures of output and expenditures) from year *t-1* to year *t* is calculated as

$$100(Q_t^F - 1.0).$$

Similarly, price indexes are calculated using the Fisher formula

$$P_t^F = \sqrt{\frac{\sum p_t q_{t-1}}{\sum p_{t-1} q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_{t-1} q_t}},$$

which is the geometric mean of a Laspeyres price index (P_t^L) and a Paasche price index (P_t^P), or

$$P_t^F = \sqrt{P_t^L \times P_t^P}\,.$$

The chain-type quantity index value for period t is $I_t^F = I_{t-1}^F \times Q_t^F$, and the chain-type price index is calculated analogously. Chain-type real output and price indexes are presented with the reference year (b) equal to 100; that is, $I_b = 100$.

The current-dollar change from year t-1 to year t expressed in the form of a ratio is equal to the product of the changes in the Fisher price and quantity indexes:

$$\frac{\sum p_t q_t}{\sum p_{t-1} q_{t-1}} = \sqrt{\frac{\sum p_t q_{t-1}}{\sum p_{t-1} q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_{t-1} q_t}} \times \sqrt{\frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}} \times \frac{\sum p_t q_t}{\sum p_t q_{t-1}}} = P_t^F \times Q_t^F\,.$$

The same formulas are used to calculate the quarterly (and for some components, monthly) chain-type indexes. All quarterly chain-type indexes for completed years that have been included in an annual or comprehensive update are adjusted so that the quarterly indexes average to the corresponding annual index. When an additional year is completed between annual updates, the annual index is computed as the average of the quarterly indexes, so no adjustment is required to make the quarterly and annual indexes consistent. For example, until the 2012 annual update was released, the chain-type indexes for the year 2011 were computed as the average of the four quarterly indexes for 2011.

Properties of chain-type measures

The chain-type indexes based on the Fisher formula have several advantages over the fixed-weighted indexes that BEA used before 1996.[16]

- They produce percent changes in quantities and prices that are not affected by the choice of the reference period.
- They eliminate the substitution bias in measures of real GDP growth that are derived using fixed-weighted indexes. This bias tends to cause an understatement

[16] For information on BEA's introduction of chain-type indexes as its featured measure of real output and prices, see J. Steven Landefeld and Robert P. Parker, "Preview of the Comprehensive Revision of the National Income and Product Accounts: BEA's New Featured Measures of Output and Prices," *Survey* 75 (July 1995): 31–38. See also J. Steven Landefeld and Robert P. Parker, "BEA's Chain Indexes, Time Series, and Measures of Long-Term Economic Growth," *Survey* 77 (May 1997): 58–68; and J. Steven Landefeld, Brent R. Moulton, and Cindy M. Vojtech, "Chained-Dollar Indexes: Issues, Tips on Their Use, and Upcoming Changes," *Survey* 83 (November 2003): 8–16.

of growth for periods before the reference year and an overstatement of growth for periods after the reference year.

- They eliminate the distortions of growth in components and in industries that result from the fixed-weighted indexes.
- They eliminate the anomalies that arise from using recent-period weights to measure periods in the past when a far different set of prices prevailed. For example, the prices of defense equipment in the 2000s are not appropriate for measuring the real changes in defense spending in the 1940s.
- They eliminate the inconvenience and confusion associated with BEA's previous practice of updating weights and years—and thereby rewriting economic history—about every 5 years.

Despite the greater accuracy provided by the chain-type indexes, users of macroeconomic statistics need more than index numbers and percent changes. The earlier fixed-weighted estimates were denominated in constant dollars, and the real levels for the components of GDP added up to total GDP. Because the system was additive, the shares of the real components reflected their relative importance in total GDP. Similarly, in decomposing total GDP growth by component, the change in constant-dollar values measured the component's contribution to the change in the fixed-weighted aggregate. For GDP and most of its components, BEA prepares estimates in chained dollars as well as chain-type indexes (see the appendix to this chapter). However, because these chained-dollar measures are not based on a single set of weights, they are not additive and thus do not yield accurate measures of shares and contributions to growth.

For real GDP and its major components, BEA provides tables that present accurate estimates of contributions to growth rates that are based on chain-type quantity indexes rather than on the chained-dollar estimates (see the appendix). In addition, BEA provides measures of percentage shares that are based on current-dollar values. Because current-dollar values provide the weights for the chain-type indexes, shares calculated from these estimates rather than from the chained-dollar estimates should be used to indicate the relative importance of components.

APPENDIX

Calculation of Output and Price Indexes

The market (and nonmarket) values used to measure GDP and the other NIPA estimates are in current dollars—that is, they represent the values of transactions taking place in the current time period. In turn, these transactions reflect a combination of physical quantities and prices. As shown in exhibit 4.1, in year 1, 10 apples at a price of $0.20 per apple can be purchased for $2.00. If the transactions in a given time period are compared with those in another time period, the differences in the current-dollar values can be attributed to differences in quantities and to differences in prices. In year 2, 20 apples at a price of $0.25 per apple can be purchased for $5.00. The increase in expenditures from $2.00 to $5.00, or 150 percent, can be separated into quantity and price elements. The quantity of apples purchased increased from 10 to 20, or 100 percent, and the price of apples increased from $0.20 to $0.25 or 25 percent.

Exhibit 4.1

Year 1			
	Expenditures	Quantity	Price
Apples	$2.00	10	$0.20
Oranges	$3.00	30	$0.10
Total fruit	$5.00		
Year 2			
Apples	$5.00	20	$0.25
Oranges	$4.00	20	$0.20
Total fruit	$9.00		

For most NIPA components, estimates of physical quantities are not available. Instead, “real” estimates—that is, estimates that exclude the effects of price change—are derived by “deflating” (dividing) the current-dollar value by appropriate price indexes. In order to prepare such estimates, a statistical application must be used that establishes a common unit price as the basis for comparison. For exhibit 4.1, one way to accomplish this is to value the second-period transaction in the price of the first period: 20 apples at the year 1 price of $0.20 is equal to $4.00, and so the real estimate increases from $2.00 in year 1 to $4.00 in year 2, or 100 percent. Alternatively, the first-period transaction could have been valued in second-period prices: 10 apples at the year 2 price of $0.25 is equal to $2.50, and so the real estimate increases from $2.50 in year 1 to $5.00 in year 2, or 100 percent.

Thus, the separation of current-dollar change into price and quantity elements for a single, detailed component is straightforward. However, for an aggregation of detailed components, price changes and quantity changes cannot be observed directly in the economy. Thus, the partitioning of the current-dollar change into price- and quantity-change elements becomes an analytic process. The price and quantity changes must be calculated, and the calculation method is determined by analytic requirements. Because of the complexity of the interactions of prices and quantities, the method of calculating real estimates for the NIPAs has evolved over time.

Estimates of real GNP and other components were introduced into the NIPAs in the early 1950s as a supplement to the current-dollar estimates. These measures were calculated by specifying a single base period set of prices and then valuing the output of all periods using those prices.

As shown in calculation 1 in exhibit 4.2 (page 4–19), which uses year 1 for valuation, the real estimate for the change in fruit from year 1 to year 2 is 20 percent. This approach, in which the real estimates are calculated moving forward from the base period, is called a "Laspeyres" quantity index. However, the results of the calculation are dependent on the choice of the base year for valuation. In calculation 2, which uses year 2 for valuation, the real estimate for the change in fruit from year 1 to year 2 is 6 percent. This approach, in which the estimates are calculated moving backward from the current period, is called a "Paasche" quantity index. Corresponding calculations can be made to produce Laspeyres and Paasche price indexes.

Before 1996, the real estimates in the NIPAs were calculated as Laspeyres quantity indexes, and the price estimates were calculated as implicit price deflators.[17] In calculation 4, the estimate for the change in the price of fruit from year 1 to year 2 is 50 percent. Note that one property of these estimates is that the index for total expenditures on fruit in year 2 ($9.00 / $5.00, or 1.800) is equal to the Laspeyres quantity index for year 2 multiplied by the Paasche price index for year 2: $1.200 \times 1.500 = 1.800$.

In 1996, BEA introduced chain-weighted indexes as its featured measure of the change in real GDP and in prices. These indexes, which are based on weights that are more appropriate to the time period being measured, significantly improved the accuracy of the NIPA estimates. The weights for these measures are calculated as the geometric mean of the calculations for the Laspeyres index and the Paasche index (in exhibit 4.1, as the square root of 1.200×1.059, or 1.127). Similarly, price measures are computed using weights calculated as the geometric mean of the calculations for the Laspeyres index and the Paasche index (in exhibit 1, as the square root of 1.700×1.500, or 1.597). Note that for the chain-type measures, the Fisher quantity index for year 2 multiplied by the Fisher price index for year 2 is also equal to the index for total expenditures on fruit in year 2: $1.127 \times 1.597 = 1.800$.

Note. The material presented in this section is based on the box "Note on Calculating Output and Prices" written by Jack E. Triplett and published in the article "Preview of the Comprehensive Revision of the National Income and Product Accounts: BEA's New Featured Measures of Output and Prices," *Survey of Current Business* 75 (July 1995): 32–33.

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[17] In the exhibit, all calculations involve only 2 years, so the Paasche price index and the implicit price deflator are equivalent.

Exhibit 4.2

Calculation 1: Laspeyres Quantity Index

Year 1 weighted quantity change measure for fruit: hypothetical expenditure on fruit in year 2 using year 1 prices, divided by actual expenditure on fruit in year 1

$$[(20 \times \$0.20) + (20 \times \$0.10)] / [(10 \times \$0.20) + (30 \times \$0.10)]$$

$$= \$6.00 / \$5.00 = 1.200$$

Calculation 2: Paasche Quantity Index

Year 2 weighted quantity change measure for fruit: actual expenditure on fruit in year 2, divided by hypothetical expenditure on fruit in year 1 using year 2 prices

$$[(20 \times \$0.25) + (20 \times \$0.20)] / [(10 \times \$0.25) + (30 \times \$0.20)]$$

$$= \$9.00 / \$8.50 = 1.059$$

Calculation 3: Laspeyres Price Index

Year 1 weighted price change measure for fruit:

$$[(10 \times \$0.25) + (30 \times \$0.20)] / [(10 \times \$0.20) + (30 \times \$0.10)]$$

$$= \$8.50 / \$5.00 = 1.700$$

Calculation 4: Paasche Price Index

Year 2 weighted price change measure for fruit:

$$[(20 \times \$0.25) + (20 \times \$0.20)] / [(20 \times \$0.20) + (20 \times \$0.10)]$$

$$= \$9.00 / \$6.00 = 1.500$$

Adjusting for quality change

Accurate price indexes are crucial for preparing accurate estimates of real GDP as well as corresponding productivity measures. The illustrations shown above assume that there are changes only in the prices and quantities of the goods or services being measured. The development of a price index becomes more complicated when the characteristics or quality of the goods or services are also changing. In these cases, the price index must isolate and measure only the price change and not the impacts of these other changes. There are several methods used to construct price indexes, and while most of these are designed to measure price change while holding quality constant, no method is perfect in every situation.[18] Traditional matched model indexes, which hold quality constant by specifying each variety in the sample and ensuring that exactly the same variety of product is sampled each period, work well with relatively standardized products. However, these indexes are less accurate when the characteristics, quality, market shares, and prices are changing rapidly. In such cases, alternative methods for quality adjustment may yield more accurate measures. The attribute-cost adjustment method, adopted recently by the Bureau of Labor Statistics (BLS) to construct the consumer price index (CPI) for computers, uses monetary values of the attributes that affect price, obtained from the original equipment manufactures or from price compiler websites, to determine appropriate quality adjustments.[19] The hedonic method uses regression analysis to determine statistical relationships between observed price changes and changes in the characteristics and qualities of the products, and these relationships are used to hold quality constant; this method is used by BLS in many of its producer and consumer price indexes and by the Census Bureau in its housing construction and sales price indexes. Both techniques have been shown to produce price indexes that are more accurate than traditional matched model indexes in cases of rapid change.[20] BEA's primary source of indexes to deflate GDP and its components is BLS, which provides detailed price indexes—including consumer price indexes, producer price indexes, and international price indexes. These indexes, as well as other indexes from BLS, the Census Bureau, and other federal agencies, all employ methodologies to adjust for quality change. In cases where quality-adjusted indexes have not been available, BEA has developed its own indexes. For example, for a number of years in the 1980s and early 1990s, BEA produced hedonic indexes for computers; as BLS began publishing quality-adjusted PPIs for computers and peripheral equipment in the 1990s, BEA adjusted its methodologies to incorporate them. In the late 1990s, BEA introduced quality-adjusted price indexes for semiconductors, for digital telephone switching equipment, and for computer software; more recently, BEA has introduced a hedonic price index for photocopying equipment.[21]

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[18] An exception is a unit value index, which measures the change in the value of items without holding characteristics, quality, or even the mix of items constant.

[19] For more information on how BLS calculates the CPI for computers, see http://www.bls.gov/cpi/cpifaccomp.htm.

[20] For more information on the hedonic method underlying producer and consumer price indexes, see http://www.bls.gov/ppi/ppicomqa.htm and http://www.bls.gov/cpi/cpihqaitem.htm and http://www.bls.gov/cpi/cpihqaitem.htm. For more information on the hedonic method underlying housing price indexes, see http://www.census.gov/construction/cpi.

[21] For more information these quality-adjustment techniques, see "Prices and Output for Information and Communication Technologies," on BEA's Website at http://www.bea.gov/national/info_comm_tech.htm.

Statistical Tools and Conventions

This section describes some of the statistical tools and conventions that BEA uses in preparing and presenting the NIPA estimates. In general, these statistical operations are used to transform the estimates into alternative formats that facilitate analytical or presentational uses.

Chained-dollar measures

As a supplement to its chain-type quantity indexes, BEA prepares measures of real GDP and its components in a dollar-denominated form, designated "chained (2009) dollar" estimates. For GDP and most other series, the chained-dollar value CD_t^F is calculated by multiplying the reference year current-dollar value $\sum p_b q_b$) by the chain-type Fisher quantity index (I_t^F) and dividing by 100. For period *t*,

$$CD_t^F = \sum p_b q_b \times I_t^F / 100 .$$

Thus, for example, if a current-dollar GDP component is equal to \$200 in 2009 and if the quantity index for this component increased 15 percent by 2012, then the chained (2009) dollar value of this component in 2012 would be \$200 x 115 ⁄ 100, or \$230.

The chained (2009) dollar estimates provide measures to calculate the percent changes for GDP and its components that are consistent with those calculated from the chain-type quantity indexes; any differences are small and due to rounding. For most components of GDP, the chained-dollar estimates also provide rough approximations of their relative importance and of their contributions to real GDP growth for years close to 2009. However, for components—such as computers and other high-tech equipment—with rapid growth in real output and sharply falling prices, the chained-dollar levels (as distinct from chain-weighted indexes and percent changes) will overstate their relative importance to GDP growth.

In addition, chained-dollar values for the detailed GDP components will not necessarily sum to the chained-dollar estimate of GDP (or of any intermediate aggregate), because the relative prices used as weights for any period other than the reference year differ from those used for the reference year. BEA provides a measure of the extent of such differences by showing a "residual" line on chained-dollar tables that indicates the difference between GDP (and other major aggregates) and the sum of the most detailed components in the table.

For periods close to the reference year, when there usually has not been much change in the relative prices that are used as the weights for calculating the chain-type index, the residuals tend to be small, and the chained (2009) dollar estimates can be used to approximate the contributions to growth and to aggregate the detailed estimates.

However, it is preferable to use estimates of exact contributions, which are described in the next section.

Some exceptions to the above methodology are made for a few components of GDP. For cases in which the components of an aggregate include large negative values, the Fisher formula cannot be used because it would require taking the square root of a negative number. In such cases, one of two other methods is used.

- Quantity estimates are calculated as the sum of, or as the difference between, chained-dollar series that measure flows. For example, real net exports is derived as the difference between real exports and real imports.
- Quantity estimates are calculated as the difference between measures of chain-weighted stocks. For example, the real annual change in private inventories is derived as the difference between real beginning-of-year inventories and real end-of-year inventories.

The inability to calculate a particular Fisher quantity index (for example, for change in private inventories) because of negative values usually does not extend to the calculation of higher level aggregates (for example, quantity indexes for gross private domestic investment and for GDP can be computed). The calculation of contributions to percent change is not affected by negative values, so they can be calculated for all components.

The chain-dollar estimates are used in the calculation of another price index, the *implicit price deflator* (IPD). The IPD_t^F for period *t* is calculated as the ratio of the current-dollar value to the corresponding chained-dollar value, multiplied by 100, as follows:

$$IPD_t^F = \frac{\sum p_t q_t}{CD_t^F} \times 100 .$$

For all aggregates and components and for all time periods, the value of the IPD is very close to the value of the corresponding chain-type price index. Note that this definition of the IPD differs from that used before the introduction of chain-type measures in1996, when the IPD was defined as the ratio of the current-dollar value to the corresponding constant-dollar value.

Contributions to percent change

As one moves further away from the reference year, the residual tends to become larger, and the chained-dollar estimates are less useful for analyses of contributions to growth. For this reason, BEA also shows contributions of major components to the percent change in real GDP (and to the percent change in other major aggregates) that use exact formulas for attributing growth.

The contributions to percent change in a real aggregate, such as real GDP, provide a measure of the composition of growth in the aggregate that is not affected by the nonadditivity of its components. This property makes contributions to percent change a valuable tool for economic analysis. The contribution to percent change ($C\%\Delta_{i,t}$) in an aggregate in period t that is attributable to the quantity change in component i is defined by the formula

$$C\%\Delta_{i,t} = 100 \times \frac{((p_{i,t} / P_t^F) + p_{i,t-1}) \times (q_{i,t} - q_{i,t-1})}{\sum_j ((p_{j,t} / P_t^F) + p_{j,t-1}) \times q_{j,t-1}},$$

where

P_t^F is the Fisher price index for the aggregate in period t relative to period $t-1$;
$p_{i,t}$ is the price of the component i in period t; and
$q_{i,t}$ is the quantity of the component i in period t.

The summation with subscript j in the denominator includes all the deflation-level components of the aggregate. Contributions of subaggregates (such as PCE goods) to the percent change of the aggregate (say, PCE or GDP) are calculated by summing the contributions of all the deflation-level components contained in the subaggregate.[22]

For annual estimates, no adjustments are required for the contributions to sum exactly to the percent change in the aggregate. For quarterly estimates, adjustments are required to offset the effects of adjustments that were made to equate the average of the quarterly estimates to the corresponding annual estimate and to express the percent change at annual rate. The same formula is used for both annual and quarterly estimates of contributions to percent change in all periods. The only variation in the method of calculation is that the annual contributions for the most recent year are based on a weighted average of the quarterly contributions until the next annual update.

Annual rates

Quarterly and monthly NIPA estimates in current and chained dollars are presented at annual rates, which show the value that would be registered if the level of activity measured for a quarter or for a month were maintained for a full year. Annual rates are used so that periods of different lengths—for example, quarters and years—may be easily compared. These annual rates are determined simply by multiplying the estimated rate of activity by 4 (for quarterly data) or by 12 (for monthly data).

[22] See Marshall B. Reinsdorf, W. Erwin Diewert, and Christian Ehemann, “Additive Decompositions for Fisher, Tornqvist, and Geometric Mean Indexes,” *Journal of Economic and Social Measurement* 28 (2002): 51–61, www.econ.ubc.ca/diewert/additive.pdf.

Growth rates

In general, percent changes in the NIPA estimates are also expressed at annual rates, which show the value that would be registered if the pace of activity measured for a time period were maintained for a full year.[23] Calculating these changes requires a variant of the compound interest formula,

$$r = \left[\left(\frac{GDP_t}{GDP_0}\right)^{m/n} - 1\right] \times 100,$$

where

r is the percent change at an annual rate;
GDP_t is the level of activity in the later period;
GDP_0 is the level of activity in the earlier period;
m is the periodicity of the data (for example, 1 for annual data and 4 for quarterly data); and
n is the number of periods between the earlier and later periods (that is, t-0).

Thus, for example, if a component increases from \$100 in the first quarter to \$105 in the second quarter (5 percent at a quarterly rate), the annual rate of increase is (($105/$100)$^{4/1}$ – 1) x 100 = 21.6 percent.

Rebasing an index

In the NIPAs, quantities and prices are generally expressed as index numbers with a reference year—at present, the year 2009—equal to 100. These indexes can easily be rebased to a different reference year without changing the relationship between the series values. To rebase, divide the entire index by the index value of the desired reference year. As illustrated in table 4.8, the original index is rebased from year 1 to year 2 by dividing each of the original index values by the index value in year 2 (for year 1, 100.0 ⁄ 110.0 = 90.9). Note that the year-to-year percent changes are unaffected by the rebasing.

Table 4.8—Example of Index Rebasing

Year	Original index	Percent change	Rebased index	Percent change
1	100.0	…..	90.9	…..
2	110.0	10.0	100.0	10.0
3	120.0	9.1	109.1	9.1
4	130.0	8.3	118.2	8.3

[23] The growth rates in the NIPA monthly series, such as personal income, are not expressed at annual rates.